Published by Institute of Physics Publishing for SISSA

Received: *February 13, 2008*
Revised: *March 17, 2008*
Accepted: *April 9, 2008*
Published: *April 14, 2008*

# Thermodynamic similarity between the noncommutative Schwarzschild black hole and the Reissner-Nordström black hole

**Wontae Kim**

*Department of Physics and Center for Quantum Spacetime, Sogang University, Seoul 121-742, Korea*
*E-mail:* wtkim@sogang.ac.kr

**Edwin J. Son**

*Department of Physics and Basic Science Research Institute, Sogang University, Seoul 121-742, Korea*
*E-mail:* eddy@sogang.ac.kr

**Myungseok Yoon**

*Center for Quantum Spacetime, Sogang University, Seoul 121-742, Korea*
*E-mail:* younms@sogang.ac.kr

Abstract: We study thermodynamic quantities and examine the stability of a black hole in a cavity inspired by the noncommutative geometry. It turns out that thermodynamic behavior of the noncommutative black hole is analogous to that of the Reissner-Nordström black hole in the near extremal limit. Moreover, we identify the noncommutative parameter with the squared electric charge with some constants.

Keywords: Black Holes, Models of Quantum Gravity, Non-Commutative Geometry.

JHEP04(2008)042

## Contents



JHEP04(2008)042

## 1. Introduction

Thermodynamics of a black hole is one of the most interesting issues in the theoretical physics. Bekenstein has suggested that the entropy of a black hole is proportional to its surface area [1] and Hawking's analysis for its origin from the point of view of quantum field theory [2] has led to the result that the black hole has a thermal radiation with the temperature $T_H = \kappa/2\pi$, where $\kappa$ is its surface gravity. Since then, there are many studies for thermodynamics in a cavity with a finite size in various black holes [3 – 9].

A complete explanation for the final state after the evaporation of the black hole is important but it has not been achieved yet since the full quantum gravity has been still unknown. However, there are two candidates for quantum gravity, which are the string theory and the loop quantum gravity. By the string/black hole correspondence principle [10], stringy effects cannot be neglected in the late stage of a black hole. In the string theory, coordinates of the target spacetime become *noncommutating* operators on a $D$-brane as $[\mathbf{x}^\mu, \mathbf{x}^\nu] = i\theta^{\mu\nu}$ [11], where $\theta^{\mu\nu}$ is an anti-symmetric matrix which determines the fundamental cell discretization of spacetime much in the same way as the Planck constant $\hbar$ discretizes the phase space. Recently, it has been shown that Lorentz invariance and unitary, raised in the Weyl-Wigner-Moyal *-product approach, can be achieved by assuming $\theta^{\mu\nu} = \theta\ \mathrm{diag}(\epsilon_1, \ldots, \epsilon_{D/2})$ [12 – 14], where $\theta$ and $D$ are a constant and the dimension of spacetime [15]. In ref. [16]. there has been the study on the thermodynamics of the Reissner-Nordström (RN) black hole, considering the effects of space noncommutativity.

In this work, we would like to study thermodynamics of a static and spherically symmetric black hole, considering the effects of noncommutative geometry. Especially, we wish to point out an analogy between the noncommutative black hole and the RN black hole, which has been commented shortly in ref. [17]. We shall show that the parameter $\theta$ in the noncommutative black hole plays a similar role with an electric charge in RN black hole. In section 2, we introduce a noncommutative black hole and examine the relation between

the mass and its horizons. Comparing the noncommutative black hole to the RN black hole, it can be shown that there is an analogy between them in section 3. In section 4, we analyze the thermodynamic properties of the noncommutative black hole in a cavity with a finite size and check the thermodynamic stability of the black hole. It can be found that these properties are similar to those of the RN black hole. Finally, some discussions are given in section 5.

## 2. Schwarzschild black hole inspired by the noncommutative geometry

We would like to examine the metric of the Schwarzschild black hole when there exists the noncommutativity of spacetime. It has been shown that noncommutativity eliminates point-like structures in favor of smeared objects in flat spacetime [12]. The effect of smearing is mathematically implemented by replacing the position Dirac-delta function with a Gaussian distribution of the width $\sqrt{\theta}$. In a static, spherically symmetric case with this logical connection, the mass density of a gravitational source is chosen as [13]

$$\rho_\theta = \frac{M}{(4\pi\theta)^{3/2}} \exp\left(-\frac{r^2}{4\theta}\right), \tag{2.1}$$

where the total mass $M$ is diffused throughout the region of linear size $\sqrt{\theta}$ and the $\theta$ is a constant parameter representing noncommutativity.

For a static and spherically symmetric metric, the density (2.1) and the conservation law tell us that the energy-momentum tensor is given by $T^\mu{}_\nu = \mathrm{diag}(-\rho_\theta, p_r, p_\perp, p_\perp)$, where the radial and the tangential pressure are given by $p_r = -\rho_\theta$ and $p_\perp = -\rho_\theta - \frac{1}{2} r \partial_r \rho_\theta$, respectively. Then, solving the Einstein equations of motion, we obtain the line element as

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2 \tag{2.2}$$

with

$$f(r) = 1 - \frac{2m(r)}{r} = 1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right), \tag{2.3}$$

where the mass distribution $m(r) = (2M/\sqrt{\pi})\gamma(3/2, r^2/4\theta)$ is straightforwardly obtained from the density (2.1), and the lower incomplete gamma function is defined by

$$\gamma(a, z) \equiv \int_0^z t^{a-1} e^{-t} dt. \tag{2.4}$$

Note that we only change the point-like structure of the Schwarzschild black hole to a smeared object so that the red-shift function (2.3) has a similar form to Schwarzschild metric except the mass distribution $m(r)$. Moreover, it can be easily checked that eq. (2.3) is reduced to the Schwarzschild metric in the limit of $r/\sqrt{\theta} \to \infty$, that is, $m(r) \to M$.

However, the presence of noncommutativity changes the Schwarzschild-like behavior into the RN-like one in the region where the noncommutativity cannot be neglected, which will be dealt with in detail in the next section. As a short preview, we see that there are two horizons, i.e., the inner (Cauchy) horizon $r_C$ and the outer (event) horizon $r_H$, and there exists the minimal mass $M_0$ below which no black hole can be formed. Those facts



JHEP04(2008)042

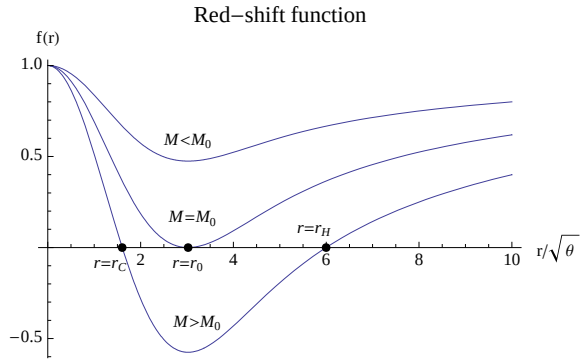


**Figure 1:** The red-shift function $f(r)$ is shown with respect to $r/\sqrt{\theta}$. There is no horizon for $M = \sqrt{\theta} < M_0$(top), while one degenerate and two horizons exist for $M = M_0 \approx 1.9\sqrt{\theta}$(middle) and $M = 3\sqrt{\theta} > M_0$(bottom), respectively.

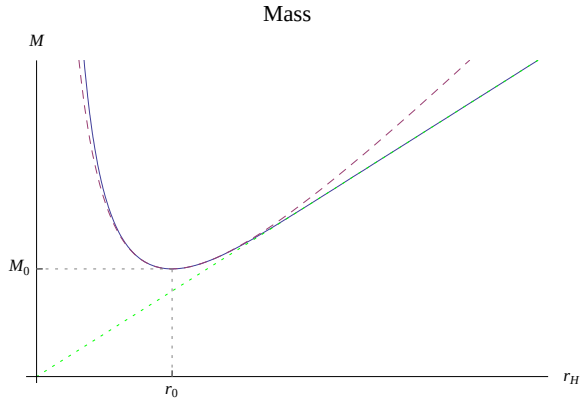


**Figure 2:** The solid, the dashed, and the dotted lines show the relations between the mass and the horizon of the noncommutative, the scaled RN, and the Schwarzschild black holes, respectively, where "scaled" means $r_H = (\alpha^2 - 1)\tilde{r}_+$ and $\tilde{r}$ indicates the radial coordinate in the RN black hole.

are seen from figure 1. Moreover, at the minimal mass $M = M_0$, the inner and the outer horizons are met at the minimal horizon $r_0$ ($r_C \leq r_0 \leq r_H$). The minimal mass is more explicitly seen in the mass relation as follows: It results from $r_H = 2m(r_H)$ that the total mass is related to the event horizon by

$$M = \frac{r_H\sqrt{\pi}}{4\gamma_H}, \tag{2.5}$$

where $\gamma_H = \gamma\left(\frac{3}{2}, \frac{r_H^2}{4\theta}\right)$. Then, the minimal mass $M_0$ is explicitly seen in figure 2.

It seems to be appropriate to note that we can hide the parameter $\theta$ in the red-shift function by redefining mass and the radial coordinate as $M \to M' = M/(2\sqrt{\theta})$ and $r \to r' = r/(2\sqrt{\theta})$ so that the function (2.3) is reduced to $f(r) = 1 - 4M'\gamma(3/2, r'^2)/(r'\sqrt{\pi})$. Then, it is deduced from the mass relation $M' = r'_H\sqrt{\pi}/[4\gamma(3/2, r'^2_H)]$ that both the radius and the mass of the minimal black hole are proportional to $\sqrt{\theta}$ and can be written as



JHEP04(2008)042

$r_0 = 2\alpha\sqrt{\theta}$ and $M_0 = \sqrt{\theta\pi}/(4\alpha^2 e^{-\alpha^2})$, where the constant $\alpha \equiv r_0'$ is determined by

$$2\alpha^3 e^{-\alpha^2} = \gamma\left(\frac{3}{2}, \alpha^2\right), \tag{2.6}$$

and we can find $\alpha \approx 1.51122$ numerically, so we get $r_0 \approx 3.02244\sqrt{\theta}$ and $M_0 \approx 1.90412\sqrt{\theta}$.

## 3. Near extremal limit and analogy with the RN black hole

In spite of the similarity of the metric (2.3) to the Schwarzschild black hole, one might think from figure 2 that the mass profile of the noncommutative black hole has a similar behavior to that of the RN black hole in the vicinity of the minimal horizon $r_0$. Since the extremal limit for the RN black hole is given by $\tilde{M} \to \tilde{Q}$ or $\tilde{r}_+ \to \tilde{Q}$, the extremal limit for the noncommutative black hole can be taken as $r_H \to r_0$, in which case $r_C$ also goes to $r_0$. So, this section is mainly devoted to the analogy with the RN black hole in the near extremal limit. At this purpose, we first recall the Hawking temperature of the RN black hole. The metric of the RN black hole is given by

$$\tilde{f}(\tilde{r}) = 1 - \frac{2\tilde{M}}{\tilde{r}} + \frac{\tilde{Q}^2}{\tilde{r}^2}, \tag{3.1}$$

where $\tilde{M}$ and $\tilde{Q}$ are the mass and the electric charge of the black hole. The inner ($r_-$) and outer ($r_+$) horizons are given by $\tilde{r}_\pm = \tilde{M} \pm \sqrt{\tilde{M}^2 - \tilde{Q}^2}$. Since the mass and the charge can be written as $\tilde{M} = (\tilde{r}_+ + \tilde{r}_-)/2$ and $\tilde{Q} = \sqrt{\tilde{r}_+\tilde{r}_-}$, we can rewrite the mass in terms of $\tilde{r}_+$ and $\tilde{Q}$, similarly to eq. (2.5),

$$\tilde{M} = \frac{1}{2}\left(\tilde{r}_+ + \frac{\tilde{Q}^2}{\tilde{r}_+}\right) \geq \tilde{M}_0 = \tilde{Q}, \tag{3.2}$$

where the equality is satisfied with $\tilde{r}_+ = \tilde{r}_0 = \tilde{Q}$. The Hawking temperature is obtained as

$$T_H^{\rm RN} = \frac{\tilde{r}_+ - \tilde{r}_-}{4\pi\tilde{r}_+^2} = \frac{\tilde{r}_+^2 - \tilde{Q}^2}{4\pi\tilde{r}_+^3}. \tag{3.3}$$

Then, the Hawking temperature of the near extremal RN black hole is written as

$$T_H^{\rm RN} \simeq \frac{\tilde{r}_+ - \tilde{Q}}{2\pi\tilde{Q}^2} + O(\tilde{r}_+ - \tilde{Q})^2. \tag{3.4}$$

On the other hand, the Hawking temperature of the noncommutative black hole is calculated as

$$T_H = \frac{1}{4\pi r_H}\left[1 - \frac{M r_H^2}{\sqrt{\pi}\theta^{3/2}} \exp\left(-\frac{r_H^2}{4\theta}\right)\right]. \tag{3.5}$$

For the limit of $r_H \gg 2\sqrt{\theta}$, it recovers the Hawking temperature of the Schwarzschild black hole $T_H = 1/(4\pi r_H)$. Now, the Hawking temperature near extremal regime is given by

$$T_H \simeq \frac{\xi(r_H - r_0)}{2\pi r_0^2} + O(r_H - r_0)^2, \tag{3.6}$$



JHEP04(2008)042

where the mass relation (2.5) is used and $\xi = \alpha^2 - 1$.

Comparing eqs. (3.4) and (3.6), one can easily find the relations between the horizons by identifying

$$r_H = \xi \tilde{r}_+, \tag{3.7}$$

$$r_0 \equiv 2\alpha\sqrt{\theta} = \xi \tilde{Q}, \tag{3.8}$$

then, eq. (3.6) becomes

$$T_H \simeq \frac{\tilde{r}_+ - \tilde{Q}}{2\pi\tilde{Q}^2} + O(\tilde{r}_+ - \tilde{Q})^2, \tag{3.9}$$

which concludes $T_H \simeq T_H^{\rm RN}$ in the leading order. Thus, the noncommutative black hole behaves similar to the RN black hole in the near extremal limit. Moreover, the noncommutative parameter is related to the charge of the RN black hole by

$$\theta = \frac{\xi^2 \tilde{Q}^2}{4\alpha^2}. \tag{3.10}$$

However, the rescaled radius $r_H = \xi\tilde{r}_+$ does not match the two minimal masses $M_0$ and $\tilde{M}_0$, since $r_0 \neq M_0$ whereas $\tilde{r}_0 = \tilde{M}_0$. In order to find out the relation between the two masses $M$ and $\tilde{M}$, we first expand eq. (2.5) in the near extremal limit,

$$M \simeq M_0 \left[1 + \xi \left(\frac{r_H - r_0}{r_0}\right)^2\right] + O(r_H - r_0)^3. \tag{3.11}$$

Next, eq. (3.2) is expanded in the near extremal limit as

$$\tilde{M} \simeq \tilde{Q} \left[1 + \frac{1}{2}\left(\frac{\tilde{r}_+ - \tilde{Q}}{\tilde{Q}}\right)^2\right] + O(\tilde{r}_+ - \tilde{Q})^3. \tag{3.12}$$

So, the two masses are related in the near extremal limit as $(M - M_0)/M_0 \simeq 2\xi(\tilde{M} - \tilde{Q})/\tilde{Q}$ in the leading order. In fact, the curve for the RN black hole in figure 2 is plotted with respect to the rescaled radius $r_H = \xi\tilde{r}_+$.

## 4. Thermodynamics of the noncommutative black hole

We now consider a cavity with the finite size $R$. The local temperature on the boundary of the cavity is given by [5]

$$T = \frac{T_H}{\sqrt{f(R)}}. \tag{4.1}$$

It can be seen from figure 3 that the local temperature of the noncommutative black hole behaves like the RN (Schwarzschild) black hole for the small (large) black hole.

The temperature has two extrema: one is the local maximum at $r_H = r_1$ and the other is the local minimum at $r_H = r_2$. There is one small black hole for $0 < T < T_2$ and one large black hole for $T > T_1$, where $T_i = T|_{r_H = r_i}$ with $i = 1, 2$. For the case of $T_2 < T < T_1$, there are three black hole solutions.



JHEP04(2008)042

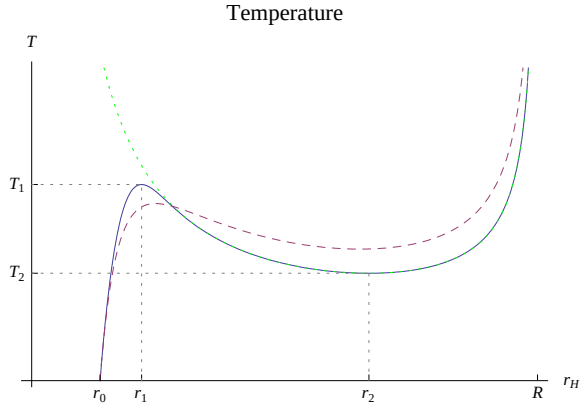


**Figure 3:** The solid, dashed, and dotted lines show the relations between the local temperature and the horizon of the noncommutative, the scaled RN, and the Schwarzschild black holes, respectively. For $R = 10$ and $\theta = 0.2$, we obtain $r_0 \approx 1.35167$, $r_1 \approx 2.17241$, $r_2 \approx 6.66667$, $T_1 \approx 0.0377456$, and $T_2 \approx 0.0206748$.

Since the entropy is proportional to the area of event horizon by

$$S = \frac{A}{4} = \pi r_H^2, \tag{4.2}$$

and the first law of thermodynamics $dE = TdS$ should be satisfied for a fixed $R$, we obtain the energy as

$$E = M_0 + \int_{S_0}^{S} TdS = M_0 + 2\pi \int_{r_0}^{r_H} r'_H T(r'_H, R) dr'_H, \tag{4.3}$$

Here, the boundary condition $E = M_0$ for $r_H = r_0$ is considered and the thermodynamic energy of the RN black hole is

$$E^{\rm RN} = \tilde{M}_0 + 2\pi \int_{\tilde{r}_0}^{\tilde{r}_+} \tilde{r}'_+ T^{\rm RN}(\tilde{r}'_+, \tilde{Q}, \tilde{R}) d\tilde{r}'_+, \tag{4.4}$$

where the local temperature of the RN black hole is written as

$$T^{\rm RN} = \frac{T_H^{\rm RN}}{\tilde{f}(\tilde{R})}. \tag{4.5}$$

Moreover, our definition of energy (4.3) is consistent with that of the Schwarzschild black hole in ref. [7] for the limit of $\theta \to 0$, and the energy is positive definite as shown in figure 4.

In order to check the stability of the noncommutative black hole, we calculate the heat capacity as

$$C_A = \left( \frac{\partial E}{\partial T} \right)_R, \tag{4.6}$$

where $A = 4\pi R^2$ is the area of the boundary of the cavity. The figure 5 shows the behavior of the heat capacity. Since the heat capacity is positive for $r_0 < r_H < r_1$ and $r_H > r_2$, the small and the large black holes are stable. In the case of $r_1 < r_H < r_2$, the black hole is



JHEP04(2008)042

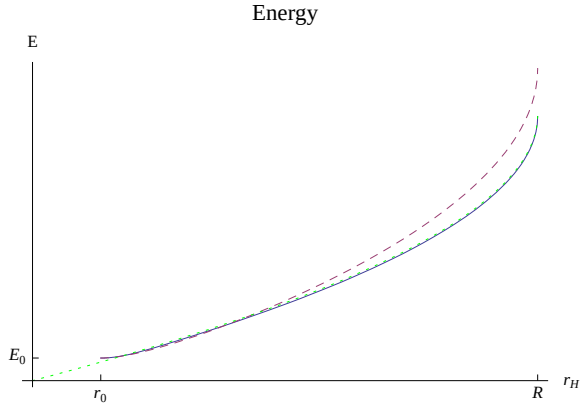


**Figure 4:** The solid, the dashed, and the dotted lines show the relations between the energy and the horizon of the noncommutative, the scaled RN, and the Schwarzschild black holes, respectively. For $R = 10$ and $\theta = 0.2$, we obtain $r_0 \approx 1.35167$ and $E_0 \approx 0.904251$.

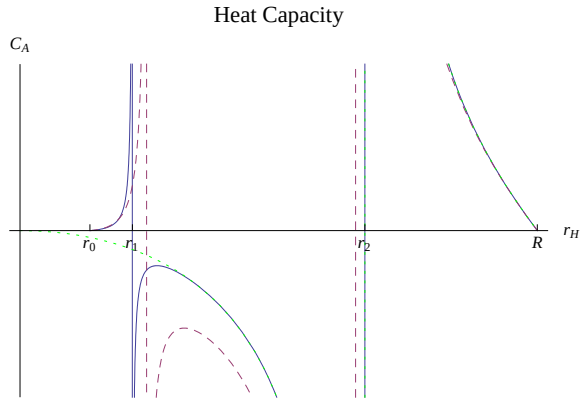


**Figure 5:** The solid, the dashed, and the dotted lines show the relations between the local temperature and the horizon of the noncommutative, the scaled RN, and the Schwarzschild black holes, respectively.

unstable since the heat capacity is negative. And the heat capacity approaches zero as $r_H$ goes to $r_0$ or $R$. This stability can be examined by considering the free energy.

The off-shell free energy of the noncommutative black hole within the cavity is given by

$$F = E(r_H, R) - TS(r_H), \tag{4.7}$$

where $E$ and $S$ are given from eqs. (4.3) and (4.2), respectively, and $T$ is an arbitrary temperature. figure 6 shows the behavior of the free energy as a function of the horizon for several temperatures. For $0 < T < T_2$ there is a small stable black hole, while there exists a large stable black hole for $T > T_1$. For the case of $T_2 < T < T_1$, the small and the large black hole are stable and the intermediate black hole is unstable. Then, the extrema



JHEP04(2008)042

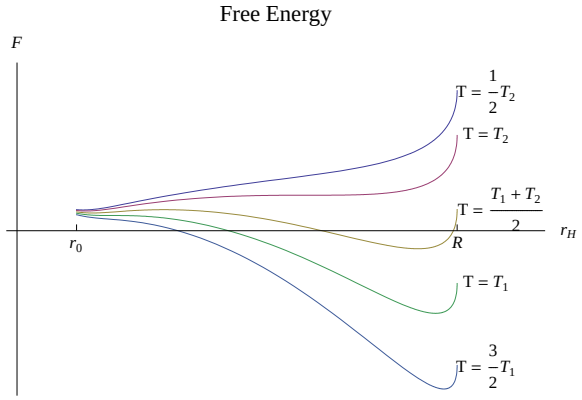


**Figure 6:** The free energy has one minimum for $0 < T < T_2$ and $T > T_1$ and three extrema for $T_2 < T < T_1$. The number of the extrema gives the number of possible black holes. The stable black holes appear when the free energy has the local minimum, while the unstable black holes appear when it has the local maximum.

of the off-shell free energy can be obtained from

$$\left(\frac{\partial F}{\partial r_H}\right)_{R,T} = 0, \tag{4.8}$$

which is nothing but $T = T(r_H)$ for a given $T$, where $T(r_H)$ is the local temperature (4.1).

## 5. Discussion

It is interesting to note that the pressure of the smeared object is negative so that it can be considered "akin" to the cosmological constant in de Sitter universe [13]. In fact, the inside of the inner horizon $r_C$ has de Sitter-like behavior, which can be seen from the line element near the origin [13, 14]. However, we are interested in the fact that the temperature of the noncommutative black hole vanishes when the horizon radius reaches the minimal horizon [18]. We have shown that the noncommutative black hole has an extremal behavior near the minimal mass, and all thermodynamic quantities are similar to those of the near-extremal RN black hole at least in the leading order.

In connection with the relations (3.7) and (3.8), the coordinates $r$ and $\tilde{r}$ are connected by $r = \xi\tilde{r}$ and the two redshift functions (2.3) and (3.1) also have similar form. To see this, we expand the functions in the near extremal limit:

$$f(r) \simeq \frac{\xi}{r_0^2}\left[(r - r_0)^2 - (r_H - r_0)^2\right] + O(r_{(H)} - r_0)^3, \tag{5.1}$$

$$\tilde{f}(\tilde{r}) \simeq \tilde{Q}^{-2}\left[(\tilde{r} - \tilde{Q})^2 - (\tilde{r}_+ - \tilde{Q})^2\right] + O(\tilde{r}_{(+)} - \tilde{Q})^3. \tag{5.2}$$

Comparing these two equations, one finds a relation $f(r) \simeq \xi\tilde{f}(\tilde{r})$ in the leading order. Then, the radial parts of the two metrics yield the line elements satisfying the relation of



JHEP04(2008)042

$ds^2 \sim dr^2/f(r) \simeq \xi d\tilde{r}^2/\tilde{f}(\tilde{r}) \sim \xi d\tilde{s}^2$, whereas the time coordinates have the same scale, $dt = d\tilde{t}$.

One might think that our result is a little dubious in the sense that general relativity breaks down and quantum gravity should be considered when the noncommutative length is identified with the Planck scale, although we have not fixed the noncommutative parameter as the Plank scale in the present calculation. To address this issue, the total mass and the radial coordinate are redefined as $M \to M' = M/(2\sqrt{\theta})$ and $r \to r' = r/(2\sqrt{\theta})$ for convenience. Then, thermal energy (4.1) and mass (2.5) can be written as $T = \mathcal{T}(r'_H, R')/\sqrt{\theta}$ and $M = \mathcal{M}(r'_H)\sqrt{\theta}$, respectively, where $\mathcal{T}$ and $\mathcal{M}$ are independent of $\theta$. To affect the back reaction of the geometry, the thermal energy should be comparable to the total mass of the black hole. So, if we assume that the locally maximal radiated energy $E = T_1 \simeq 0.015/\sqrt{\theta}$ at $r_H = r_1 \simeq 4.8\sqrt{\theta}$ in figure 3 is assumed to be equal to the total mass $M(r_H = r_1) \simeq 2.4\sqrt{\theta}$, then it can be shown that the quantum back-reaction cannot be neglected for this non-extremal black hole since the noncommutative length should be $\sqrt{\theta} \sim \ell_{\rm Pl} \sim 10^{-34}$ cm [13]. This calculation has been done for the large cavity size $R \gtrsim 100\sqrt{\theta}$. Therefore, as expected, the back-reaction effect can not be neglected when the noncommutative parameter is identified with the Plank constant since at this scale the radiated energy is comparable to the size of the black hole. Now, we want to investigate the above possibility in the near extremal limit which is a main part of the present work. Note that in this limit $\mathcal{T} \sim \epsilon r_0^2/R^2$ in the leading order, and $\mathcal{M}$ is order of 1, where $\epsilon = (r_H - r_0)/r_0 \ll 1$ and $R \gg r_0$. Thus, at the Planck scale $\sqrt{\theta} \sim \ell_{Pl}$, the radiated energy is calculated as $T \sim \epsilon(r_0/R)^2 M \ll M$. It means that the quantum back-reaction can be suppressed at the Plank scale of the noncommutativity parameter as well as at the scale $\sqrt{\theta} \gg \ell_{Pl}$ since the radiated energy is small compared to the size of the black hole. In the end, the noncommutativity in the near extremal limit cools down the black hole so that quantum back-reaction may be suppressed at least in this thermodynamic analysis. Of course, the complete answer is not yet known because of absence of the consistent quantum gravity.

Finally, we would like to make a couple of comments why this work is interesting. The noncommutativity appears in the D-brane in the string theory and its intriguing spacetime structure seems to be very special. However, this kind of noncommutativity also appears in the model of a very slowly moving charged particle on the constant magnetic field [19] and the Chern-Simon's theory [20]. In these regards, some noncommutative properties happen in various models. As we discussed so far, it appears in the near extremal RN black hole in the thermodynamic analysis if the noncommutative parameter is identified with the squared electric charge with some constants. Moreover, the metric of the near extremal noncommutative black hole has been explicitly identified with that of the near extremal RN black hole. So, we hope various properties of the noncommutative black hole along with the thermodynamic similarity can be studied in terms of the near extremal RN black hole. Conversely speaking, the near extremal RN black hole which has been widely studied in the black hole physics shares some properties of the near extremal noncommutative black hole. On the other hand, the noncommutative black hole is nonsingular and has de Sitter-like geometry near center [13]. This is the key to the thermodynamic analogy



JHEP04(2008)042

between noncommutative black holes and RN black holes in the near extremal limit, since de Sitter-like geometry gives inner (Cauchy) horizon in addition to the outer (event) horizon. In fact, de Sitter core can be seen in most regular black holes [21], which have asymptotic Schwarzschild geometry. They are also expected to be thermodynamically similar to the RN black hole in the near extremal limit. Furthermore, it would be interesting to compare the statistical entropy between the noncommutative black hole and the RN black hole in the near extremal limit.

## Acknowledgments

W. Kim and M. Yoon were in part supported by the Science Research Center Program of the Korea Science and Engineering Foundation through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number R11-2005-021. And W. Kim and E. J. Son were in part supported by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government(MOST) (R01-2007-000-20062-0).

## References

[1] J.D. Bekenstein, *Black holes and the second law*, *Lett. Nuovo Cim.* **4** (1972) 737; *Black holes and entropy*, *Phys. Rev.* **D 7** (1973) 2333; *Generalized second law of thermodynamics in black hole physics*, *Phys. Rev.* **D 9** (1974) 3292.

[2] S.W. Hawking, *Particle creation by black holes*, *Commun. Math. Phys.* **43** (1975) 199 [*Erratum ibid.* **46** (1976) 206].

[3] G.W. Gibbons and S.W. Hawking, *Action integrals and partition functions in quantum gravity*, *Phys. Rev.* **D 15** (1977) 2752.

[4] S.W. Hawking and W. Israel, *General relativity*, Cambridge University Press, Cambridge U.K. (1979).

[5] J.D. Brown, J. Creighton and R.B. Mann, *Temperature, energy, and heat capacity of asymptotically anti-de Sitter black holes*, *Phys. Rev.* **D 50** (1994) 6394 [gr-qc/9405007].

[6] B. Allen, *Euclidean Schwarzschild negative mode*, *Phys. Rev.* **D 30** (1984) 1153.

[7] J.W. York, *Black-hole thermodynamics and the euclidean Einstein action*, *Phys. Rev.* **D 33** (1986) 2092.

[8] B.F. Whiting and J.W. York, *Action principle and partition function for the gravitational field in black-hole topologies*, *Phys. Rev. Lett.* **61** (1988) 1336.

[9] D.Brown, *Black hole thermodynamics in a box*, gr-qc/9404006.

[10] L. Susskind, *String theory and the principle of black hole complementarity*, *Phys. Rev.* **D 71** (1993) 2367.

[11] E. Witten, *Bound states of strings and p-branes*, *Nucl. Phys.* **B 460** (1996) 335;
N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *JHEP* **09** (1999) 032.

[12] A. Smailagic and E. Spallucci, *Feynman path integral on the noncommutative plane*, *J. Phys.* **A 36** (2003) L467 [hep-th/0307217]; *UV divergence-free QFT on noncommutative plane*, *J. Phys.* **A 36** (2003) L517 [hep-th/0308193].



JHEP04(2008)042

[13] P. Nicolini, A. Smailagic and E. Spallucci, *Noncommutative geometry inspired Schwarzschild black hole*, *Phys. Lett.* **B 632** (2006) 547 [gr-qc/0510112].

[14] S. Ansoldi, P. Nicolini, A. Smailagic and E. Spallucci, *Noncommutative geometry inspired charged black holes*, *Phys. Lett.* **B 645** (2007) 261 [gr-qc/0612035].

[15] A. Smailagic and E. Spallucci, *Lorentz invariance, unitarity in UV-finite of QFT on noncommutative spacetime*, *J. Phys.* **A 37** (2004) 1 [*Erratum ibid.* **A 37** (2004) 7169].

[16] K. Nozari and B. Fazlpour, *Reissner-Nordstrom black hole thermodynamics in noncommutative spaces*, gr-qc/0608077.

[17] Y.S. Myung, Y-W. Kim and Y-J. Park, *Quantum cooling evaporation process in regular black holes*, *Phys. Lett.* **B 656** (2007) 221 [gr-qc/0702145].

[18] Y.S. Myung, Y-W. Kim and Y-J. Park, *Black hole thermodynamics with generalized uncertainty principle*, *Phys. Lett.* **B 645** (2007) 393 [gr-qc/0609031].

[19] W.T. Kim and J.J. Oh, *Noncommutative open strings from Dirac quantization*, *Mod. Phys. Lett.* **A 15** (2000) 1597 [hep-th/9911085].

[20] S. Deser, R. Jackiw and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys. (NY)* **140** (1982) 372 [*Erratum ibid.* **185** (1988) 406] [*Ann. Phys. (NY)* **281** (2000) 409].

[21] J.M. Bardeen, *Non-singular general-relativistic gravitational collapse*, in *Proceedings of the International Conference GR5*, Tbilisi, USSR (1968);
S. Ansoldi, *Spherical black holes with regular center: a review of existing models including a recent realization with Gaussian sources*, arXiv:0802.0330, and references therein.

JHEP04(2008)042